Cosmological Birefringence and the Microwave Background

Nathan F. Lepora
King's College, Cambridge University, Cambridge, CB2 1ST, England
(February 7, 2008)

We show that significant anisotropy in electromagnetic propagation generates a distinctive signature in the microwave background. The anisotropy may be determined by looking at the cross correlator of the E-mode and B-mode polarisation spectrum.

Linearly polarised light travels through a birefringent Universe the same as it would through a birefringent crystal; the angle of polarisation rotating dependant upon its direction of propagation. As such it represents a deviation from the Cosmological principle — where the large scale Universe is postulated to be homogenous and equivalent in all directions — with the Universe possessing a fundamental spatial anisotropy attributed to the properties of electromagnetic propagation.

Theoretically, probably the best motivation for such an anisotropy would be a small homogenous background torsion component to the background geometry of the Universe, as pointed out by Dobado and Maroto [1]. Torsion couples into the Dirac equation through minimal coupling in the covariant derivative. Regularisation of the full electromagnetic-fermion Lagrangian then produces an effective birefringence for propagating electromagnetic radiation [1]. This birefringence takes the form of a dipole for the rotation of polarisation

$$\beta(\mathbf{r}) = \frac{d}{2\Gamma_s}\hat{\mathbf{r}}\cdot\hat{\mathbf{s}}, \tag{1}$$

from polarised light emitted at light distance d, direction $\hat{\mathbf{r}}$. The axis $\hat{\mathbf{s}}$ and length scale Γ_s are related to the spatial component of the con-torsion vector $S_\alpha = \epsilon_{\alpha\beta\gamma\delta}T^{\beta\gamma\delta}$ by

$$\Gamma_s^{-1}\hat{\mathbf{s}} = -\frac{\alpha Q^2}{6\pi}\mathbf{S}, \tag{2}$$

with α the fine structure constant, $\hat{\mathbf{s}}$ a unit vector and Q^2 the total squared fermionic charge. For the Standard Model $Q^2 = 8$; other models lead to different values, for examples the supersymmetric standard model has $Q^2 = 12$. On a heuristic level it is not surprising that torsion gives this effect because of the association with rotation, first described by Cartan in 1922 [2].

Interest in birefringence has been revived recently because of Nodland and Ralston's claim to have measured a Hubble distance scale dipolar birefringence in the synchrotron radiation of radio galaxies [3]. They claim to have obtained, to about 0.1% significance, $\Gamma_s \sim 0.1cH_0^{-1}$ and $\hat{\mathbf{s}} = (0^\circ \pm 20^\circ\mathrm{decl}, 21 \pm 2hrs\mathrm{R.A.})$. Curiously, within the limits of experimental error $\hat{\mathbf{s}}$ coincides with both the dipole axis of the microwave background [4] and the rotation axes of galaxies in the Perseus-Pisces super-cluster [5]. We should point out that historically this claim predates Dobado and Morato's paper, which provided a theoretical explanation with a con-torsion of magnitude about 10^{-30}eV.

Their analysis has received some criticism [6,7], which they in turn refute [8]. The major criticism, made by several independent groups, is that their choice of statistics was intrinsically biased towards producing birefringence [7]. Different statistics indicate significant birefringence is not supported for length scales $\Gamma_s \lesssim 0.4cH_0^{-1}$.

The purpose of this letter is to point out that if large scale birefringence exists then the effects of it should be present in the radiation of the microwave background. In the near future it should be possible look for this effect, which should present a very distinctive signature onto the microwave background polarisation spectrum.

Essentially the point is that the temperature variations of the microwave background originate in the metric perturbations within the surface of last scattering. These metric perturbations also cause potential flows of matter, which gives rise to local partial polarisations of the microwave background by Thompson scattering. These polarisations split into contributions respectively uncorrelated and correlated with the temperature fluctuations, with the correlated component forming distinctive patterns around the temperature fluctuations [9]. However, if there is appreciable anisotropy in the polarisation propagation then the orientation of our measured polarisations will be skewed relative to the temperature fluctuation map. That is the basis of this letter.

Specifically, we shall discuss the effects of birefringence on *scalar* perturbations, before briefly mentioning the effect on vector and tensor modes at the end of this letter. Before discussing the effects it is necessary to discuss the polarisation spectra in more depth. For more details we refer the reader to the excellent review article by Hu and White [10].

Microwave background polarisation is induced by the last Thompson scattering of a decoupling photon. The cross section depends upon the incident and scattered polarisations $\hat{\mathbf{e}}$ and $\hat{\mathbf{e}}'$ as

$$\frac{d\sigma_T}{d\Omega} \propto |\,\hat{\mathbf{e}}\cdot\hat{\mathbf{e}}'\,|^2, \tag{3}$$

peaking for perpendicular scattering with parallel incident and scattered polarisations. The photon field is tightly bound to the baryons, forming a net photon-baryon fluid, with the incident photon field depending upon the potential flow within the last scattering surface

of last scattering. Symmetry means that local isotropic or dipolar flows produce no net effect, since scattered polarisations cancel. Thus the lowest order non-trivial local moment of the flow is quadrupolar, producing linear polarisation directed along the compressional axis of the quadrupole.

Consider a rotationally symmetric perturbation within the last scattering surface. If the perturbation represents a potential well, corresponding to a temperature cold spot, the photon-baryon fluid flows radially inwards giving azimuthally orientated local quadrupole moments; this produces an azimuthal polarisation pattern. Alternatively, for potential hill perturbations, corresponding to hot spots, the quadrupoles are rotated though ninety degrees producing a radial polarisation pattern. Considering the magnitude of polarisation, single valuedness forces the polarisation to vanish in the centre of the perturbation, with the magnitude increasing radially outwards then falling off as the gradient flow decreases. One should note that the two polarisations are orientated respectively parallel and normal to the direction of maximal polarisation gradient. This is a generic feature of scalar potentials.

The above argument is, of course, an idealisation and generally one expects the measured polarisations be dominated by random fluctuations. These fluctuations obscure the temperature correlated component. Statistically, however, one still expects some correlation with the temperature anisotropies at a level of about 15% [9].

Now consider a temperature-polarisation map of the microwave background obtained directly at the surface of last scattering for those photon that will propagate to us. Temperature represents a scalar function distributed across the surface of last scattering $T(\theta, \phi)$, whilst polarisation is a vector function $\mathbf{P}(\theta, \phi)$ such that $\mathbf{P} \cdot \hat{\mathbf{r}} = 0$. Because polarisation is $\mathbf{C}P^1$ valued $\pm\mathbf{P}$ are identified. As mentioned above only about 15% of this polarisation represents a component correlated with the temperature.

The photon field then propagates to us, red shifting as the Universe expands. If contaminatory effects, such as obscuration by our galaxy, effects of intervening galaxies and reionisation are successfully subtracted, then there are left two modifying features of the homogenous large scale geometry :

(i) Curvature of the intervening Universe will effect the angular scale of perturbations in T and $\mathbf{P}$. However, this curvature will be determined by the effects on the acoustic peak spectrum, and may be consistently taken into account.

(ii) Birefringence, if present, will alter the polarisation spectrum. Generally, the magnitude of the polarisation field $|\,\mathbf{P}\,|$ will be unaffected, whilst the direction will be rotated by an angle $\beta(\theta, \phi)$; the measured polarisation will then take the form

$$\mathbf{P}_{\mathrm{bir}}(\theta, \phi) = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \mathbf{P}. \tag{4}$$

If birefringence originates via a small homogeneous torsion, and we choose spherical polar coordinates such that the axis $\theta = 0$ coincides with the con-torsion, then the birefringence measure takes the form $\beta(\hat{\mathbf{r}}) = (d_{\mathrm{rec}}/2\Gamma_s)\cos\theta$. Here d_{rec} is the light distance to the surface of last scattering taken to good approximation to be the current horizon distance

$$d_{\mathrm{rec}}(\Omega_0) = \begin{cases} \frac{1}{\sqrt{\Omega_0 - 1}} \cos^{-1}\left(\frac{2}{\Omega_0} - 1\right) cH_0^{-1} & \Omega_0 > 1 \\ 2cH_0^{-1} & \Omega_0 = 1 \\ \frac{1}{\sqrt{1-\Omega_0}} \cosh^{-1}\left(\frac{2}{\Omega_0} - 1\right) cH_0^{-1} & \Omega_0 < 1 \end{cases} \tag{5}$$

When measuring the polarisation of the microwave background it is convenient to express it as components of the local parity eigenstates. These correspond to the E-mode, the $E > 0$ ($E < 0$) component perpendicular (parallel) to the maximal gradient of $|\,\mathbf{P}\,|$; and the B-mode, at 45° to the E-mode component. These are parity eigenstates because if one imagines a homogenous distribution of E or B-mode polarisation on a sphere then the configurations are respectively parity even and parity odd. A basis of E-mode and B-mode polarisation vectors is particularly useful because it neatly separates out perturbations with particular parity signatures: for scalar perturbations parity forces the polarisations *to be purely E-mode*. This observation is the basis of the following discussion.

The point is that birefringence converts E-mode polarisation into B-mode polarisation. E-mode polarisations within the last scattering surface, $P^E_{\mathrm{l.s.}}$, from different regions on the sky will experience conversion into both B-mode and E-mode polarisations such that

$$\begin{pmatrix} P^E \\ P^B \end{pmatrix} = \begin{pmatrix} \cos 2\beta & -\sin 2\beta \\ \sin 2\beta & \cos 2\beta \end{pmatrix} \begin{pmatrix} P^E_{\mathrm{l.s.}} \\ 0 \end{pmatrix}, \tag{6}$$

with $\beta(\theta, \phi)$ representing the amount of birefringent rotation experienced by a photon emitted from (θ, ϕ) on the sky. The angle 2β originates from the non-orthogonality of the polarisation basis.

Assuming $\beta(\theta, \phi)$ does not vary substantially on a small scale, say λ°, then we may bin the sky intoareas of about $\lambda^\circ \times \lambda^\circ$ with approximately constant β within each. Denoting the nth such region by (θ_n, ϕ_n) and the correlator over that region by $\langle\cdot\cdot\rangle_n$, Eq. (6) gives the EB cross correlator on the region n to be

$$\langle P^E P^B \rangle_n = -\frac{1}{2} \sin 4\beta(\theta_n, \phi_n) \langle P^E_{\mathrm{l.s.}} P^E_{\mathrm{l.s.}} \rangle_n. \tag{7}$$

Assuming all polarisation is from scalar modes, $\langle P^E_{\mathrm{l.s.}} P^E_{\mathrm{l.s.}} \rangle_n = \langle P^2 \rangle_n$; then the birefringence on a scale λ° is

$$\beta(\theta_n,\phi_n) = -\frac{1}{4}\sin^{-1}\left(2\frac{\langle P^E P^B\rangle_n}{\langle P^2\rangle}\right). \qquad (8)$$

Both correlators $\langle P^E P^B\rangle_n$ and $\langle P^2\rangle_n$ are determinable from a polarisation map of sufficient accuracy and resolution, with

$$\langle P^2\rangle_n = \langle P^E P^E\rangle_n + \langle P^B P^B\rangle_n + \sqrt{2}\langle P^E P^B\rangle_n. \qquad (9)$$

From an experimental point of view it is probably best to pick λ to be about the largest scale at which β is approximately constant to obtain the best statistics. We should also mention that the above relation does not require an all sky coverage and good determinations could be made from a local region of the sky.

Consider again birefringence originating from a small homogenous con-torsion field. For sufficient birefringence one would see alternating parallel stripes of E-mode and B-mode polarisations forming concentric circles on the sky, with the centres coincident in the direction of con-torsion. On stripes of maximal E-mode the B-mode is negligible and *vice-versa*. Between stripes the polarisation continuously interpolates between one mode and the other. The number of stripes, depends upon the magnitude of the birefringence, and a simple calculation gives n stripes for $\Gamma_s \approx 2d_{\rm rec}/n$. For the birefringence obtained by Nodland and Ralston this translates to 20 stripes in a critical density Universe. For the lower bounds on birefringence obtained by other authors one obtains $n \lesssim 5$.

For a sufficiently small con-torsion such that $\Gamma_s \gtrsim 2d_{\rm rec}$, a pattern of stripes is not obtained. Instead perpendicular to the con-torsion the polarisation is purely E-mode, interpolating to a mixture of E-mode and B-mode polarisation at the poles.

In addition to producing significant E-B cross correlation, birefringence should also affect the E-mode polarisation multipole spectrum. The polarisation anisotropy spectrum is described by the multipole expansion of the two point correlation function

$$C^E(\vartheta) = \langle P^E(\hat{\mathbf{r}})P^E(\hat{\mathbf{r}}')\rangle_{\hat{\mathbf{r}}\cdot\hat{\mathbf{r}}'=\cos\vartheta} \qquad (10)$$
$$= \frac{1}{4\pi}\sum_l (2l+1)C_l^E P_l(\cos\vartheta), \qquad (11)$$

with the coefficients C_l^E extractable from observations. An analogous expression represents $C^B(\vartheta)$ in terms of its coefficients C_l^B. Representing the initial coefficients at the last scattering surface by $(C_l^E)_{\rm l.s.}$, Eq. (6) implies that the effect of birefringence on the multipole expansion is to perform a linear transformation upon the coefficients. After birefringence present day coefficients are linearly related to the coefficients at last scattering by

$$C_l^E = A_{lm}(C_l^E)_{\rm l.s.}, \qquad (12)$$
$$C_l^B = B_{lm}(C_l^E)_{\rm l.s.}, \qquad (13)$$

with

$$A_{lm} = -\frac{1}{2\pi N_{lm}}\int_{\varphi=0}^{2\pi}\int_{\vartheta=0}^{\pi} P_l(\cos\vartheta)P_m(\cos\vartheta)\cos 2\beta(\vartheta,\varphi)\sin\vartheta\mathrm{d}\vartheta\mathrm{d}\varphi$$
$$B_{lm} = -\frac{1}{2\pi N_{lm}}\int_{\varphi=0}^{2\pi}\int_{\vartheta=0}^{\pi} P_l(\cos\vartheta)P_m(\cos\vartheta)\sin 2\beta(\vartheta,\varphi)\sin\vartheta\mathrm{d}\vartheta\mathrm{d}\varphi$$

and normalisation $N_{lm} = \sqrt{(2l+1)(2m+1)}$. Relations (12, 13) and (14, 15) extract the last scattering polarisation from the measured polarisation. Were birefringence to be detected, comparison to the cosmological model prediction for $(C_l^E)_{\rm l.s.}$ would offer a useful consistency check on the results.

Again we illustrate the above with a small homogenous con-torsion. In this case the linear transformations A_{lm} and B_{lm} take the form

$$A_{lm} = -\frac{1}{N_{lm}}\int_{x=-1}^{1} P_l(x)P_m(x)\cos nx\mathrm{d}x, \qquad (16)$$
$$B_{lm} = -\frac{1}{N_{lm}}\int_{x=-1}^{1} P_l(x)P_m(x)\sin nx\mathrm{d}x, \qquad (17)$$

with $n \approx 2d_{\rm rec}/\Gamma_s$ assumed to of the order one or greater. These lead to modification of the polarisation spectrum after birefringence, which conveniently splits into two contributions. Firstly, correlation of the stripes produces a new peak in the correlation function at scales $l \sim n$. Secondly, rotation of E-mode into B-mode should approximately half the total power in the E-mode polarisation spectrum. However, one should note that a new low l peak would be obscured by the reionisation peak, produced by reionisation of the Universe at red shifts $z \sim 5-20$.

Summing up, significant birefringence of a magnitude below the current experimental bounds would lead to a distinctive modification of the microwave background polarisation spectrum. Its signature would be conversion of E-mode polarisation into B-mode polarisation, with a pattern relating to the form of the birefringence. Some modification of the E-mode power spectrum would also occur, offering a useful consistency check on such an effect. If birefringence is significant in the post photon-baryon coupled Universe then its effect will be seen in the next generation of microwave background experiments.

We finish on a few points that warrant further note:
(i) Although we have concentrated specifically on illustrating birefringence with a model of homogenous torsion, it is likely that other reasonable models should lead to similar consequences. Particularly compelling is a model with an axionic condensate whose density varies linearly across the horizon [11], as predicted in some texture models. Such a model would give rise to a dipolar birefringence. Additionally the matter gradient would orientate the birefringence axis with the dipole axis of the microwave background.
(ii) Birefringence would also effect any vector or tensor

perturbations. Vector perturbations produce mainly B-mode polarisations, so birefringence would convert this into E-mode polarisation. Tensor perturbations, such as those from gravitational waves, give a mixture of E and B-mode polarisation; birefringence should mix these contributions. However, it is generally expected that scalar perturbations dominate the density spectrum.

(iii) We have assumed that for scalar potentials the initial polarisations are purely E-mode. Torsion would alter the parity properties of the potential, producing some initial B-mode polarisation. However, this contribution should be small since the comparitive length scales between birefringence and the perturbations differ by several orders of magnitude.

(iv) Although we have presented torsion as a reasonable theoretical motivation for birefringence, the absence of detectable birefringence in the microwave background would constitute an upper bound upon torsion for cosmologically relevant distance scales. The expected sensitivity should be around 10^{-32}eV; several orders of magnitude better than local torsion estimates, whose best current upper bound limits the local torsion to less than 10^{-18}eV [12].

(v) It should be mentioned that it would be difficult to reconcile a significant torsion with a conventional spin density source. Assuming that particles have spin approximately $\hbar$, a torsion around 10^{-31}eV would require a number density about 10^{40}cm^{-3}, far in excess of any conventional particle densities. Observation of a relevant birefringence would necessitate some rethinking of how torsion is sourced.

ACKNOWLEDGMENTS

I acknowledge King's College, Cambridge, for a junior research fellowship and thank P. Saffin, N. Turok and J. Weller for their valuable advice relating to this work.

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